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An investigation into the possible effects of inelastic inter-Landau level scattering on the resistivity and thermopower of a two-dimensional electron gas

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Abstract. The paper presents some results of an investigation into the quantum oscillations in the resistivity and thermopower of a 2D electron gas at low magnetic fields in the liquid ^{4}He temperature range. The thermopower is expected to be controlled by electron–phonon scattering in this temperature range and it should be a useful probe of the different effects due to inelastic intra- and inter-Landau level scattering. We do see many new phenomena which can be explained in terms of the transition from the one type of scattering to the other as the temperature and field are changed. We also see some new effects in the resistivity which may also be due to electron–phonon scattering.

1. Introduction

The first evidence that inelastic inter-Landau level scattering can play a role in quantum oscillatory effects seems to be that of Kent *et al* (1988) (see also Challis *et al* 1989, Hardy *et al* 1989). They found that when a Si MOSFET was placed in a magnetic field B, the two-dimensional electron gas (2DEG) scattered phonons in a heat pulse with an oscillatory dependence on B but with a phase that depended on $kT_{\mathrm{ph}}/\hbar\omega_{\mathrm{c}}$, where T_{ph} is the temperature of the heat pulse and $\hbar\omega_{\mathrm{c}}$ the Landau level spacing. At $kT_{\mathrm{ph}}/\hbar\omega_{\mathrm{c}} \sim 0.35$ a phase reversal was found in the quantum oscillations. The authors argued that for $kT_{\mathrm{ph}}/\hbar\omega_{\mathrm{c}} < 0.35$ intra-Landau level scattering is dominant so the oscillations are in phase with those in the electronic density of states at the Fermi level ε_{F}, and hence also with those in the resistivity (which we denote by $\tilde{\rho}_{xx}$). For $kT_{\mathrm{ph}}/\hbar\omega_{\mathrm{c}} > 0.35$ the dominant mechanism becomes inter-Landau scattering which is maximized when the Landau levels are equispaced about ε_{F} and so the phase reversal occurs.

Inelastic inter-Landau level scattering was also invoked by Leadley *et al* (1989a) to explain some unexpected results concerning the behaviour of $\tilde{\rho}_{xx}$ in heterojunctions when two electric subbands are occupied. The authors found that the amplitude of the resistivity oscillations due to the lower subband electrons is modulated by the passage of the upper subband Landau levels through ε_{F}. This in itself is not surprising because intersubband electronic scattering would be expected to give such an effect. The unexpected feature was the fact that the modulation became stronger at higher temperatures which led to the suggestion that inelastic phonon scattering of electrons between Landau levels in the two subbands might be responsible. Since then Coleridge (1990) has shown that elastic electron–impurity scattering between the levels might be able to account for this behaviour; this explanation in terms of elastic scattering appears to be more consistent with the fact that except for in the very highest mobility samples (> 100 m^2 V^{-1} s^{-1}) the zero-magnetic-field resistivity is almost completely determined by such scattering in the ^{4}He temperature range.

In contrast to the resistivity, the thermopower in the ^{4}He temperature range is expected to be almost completely controlled by electron–phonon scattering and so should be an ideal property with which to study intra- and inter-Landau level inelastic scattering. At low $kT/\hbar\omega_c$ (where T is now the temperature of both the lattice and the 2DEG) inter-Landau level scattering is not possible and so intra-Landau level scattering must be the dominant mechanism (Lyo 1989, Kubakaddi *et al* 1989). As $kT/\hbar\omega_c$ is increased inter-Landau level scattering becomes possible and could conceivably become dominant, particularly if the Landau level width is small which will serve to inhibit intra-Landau level scattering; one would then expect a phase reversal of the oscillations as in the heat pulse experiments. Since the observations of Kent *et al* (1988) were published, we have examined the quantum oscillations in the thermopower of a number of GaAs/GaAlAs heterojunctions in which only the lower subband was occupied but were unable to find any convincing evidence for a phase reversal near $kT/\hbar\omega_c \sim 0.35$ (unpublished). We always found that the thermopower oscillations, which we denote by $\tilde{S}_{xx}$, decrease faster (as a function of $kT/\hbar\omega_c$) than the equivalent oscillations in $\tilde{\rho}_{xx}$; by $kT/\hbar\omega_c \sim 0.4$, $\tilde{S}_{xx}$ has essentially disappeared whereas $\tilde{\rho}_{xx}$ is usually visible until $kT/\hbar\omega_c \sim 0.6$. Of course the rapid decay of $\tilde{S}_{xx}$ might be caused, at least in part, by an increasing out-of-phase component due to inter-Landau level scattering, but we were unable to see any resurgence of the oscillations with opposite phase at higher $kT/\hbar\omega_c$ as convincing evidence of this.

The present paper reports similar thermopower results but on a high mobility sample which can be taken from single- to double-band occupancy by photo-illumination. When two bands are occupied, we see very strong amplitude modulation effects and phase shifts in $\tilde{S}_{xx}$. Some of these effects have no counterpart in data on $\tilde{\rho}_{xx}$ taken under the same conditions of $kT/\hbar\omega_c$ and we take them as evidence for inelastic inter-Landau level effects. We also see phase shifts in $\tilde{S}_{xx}$ when only a single subband is occupied, which are also presumed to be caused by inter-Landau level scattering.

Although the present data on $\tilde{\rho}_{xx}$ were taken primarily to compare with $\tilde{S}_{xx}$, they do provide more experimental information about the amplitude modulation and reveal some new features which do not seem to be consistent with Coleridge's model and may well indicate the presence of inelastic effects. Nevertheless there is a range of $kT/\hbar\omega_c$ where Coleridge's ideas seems to be applicable and they are probably appropriate to the conditions normally encountered experimentally.

After briefly explaining the experimental methods and analysis in section 2, we outline the main features of the experimental results in section 3, and attempt to interpret these features within the context of elastic and inelastic electronic scattering in section 4.

2. Experimental technique and analysis

The heterojunction sample we have used was prepared at Philips Research Laboratories, Redhill, UK and identified by them as G590. Some details of its structure and some other properties are given by Fletcher *et al* (1991). The unilluminated sample has a carrier density of 3.9×10^{15} m^{-2} and mobility of 48 m^2 V^{-1} s^{-1}. With illumination from a red diode ε_F enters the second subband at a carrier density near 4.7 $\times 10^{15}$ m^{-2}, and the total carrier density n_T finally saturates at 6.6×10^{15} m^{-2} at which point the second subband density is about 0.3×10^{15} m^{-2}.

All of the data reported here were taken with a magnetic field $\boldsymbol{B}$ perpendicular to the 2DEG, to an accuracy of 2–3°. One obtains the best signal to noise for the thermopower by using the maximum possible temperature gradient. This was usually achieved by cooling to about 1.1 K and applying sufficient power to the heater on the free end of the sample to raise the average temperature to any desired value in the range 1.5–4.0 K. Temperature gradients were measured by carbon thermometers and hence the absolute amplitude of the thermoelectric oscillations could be determined. We also required data on the resistivity at various temperatures so these were taken during the initial cooldown.

A particular problem that we encountered was that n_T tended to decrease slightly in the period following illumination. For $n_T \sim 4.9 \times 10^{15}$ m^{-2}, n_T dropped by about 0.5% over the first hour after illumination and became stable to 0.1% only after several hours. This was a problem because we were interested in the relative phases of the oscillations observed in the resistivity and thermopower at various temperatures and a complete data set required many hours of experiments. To make absolutely sure that changes in n_T caused no spurious phase differences, in one particular experimental run we took data for which there was always a 10–12h delay between illumination and experiments.

Another difficulty that might arise in comparing the phases of resistivity and thermopower oscillations is that thermopower is essentially a two-terminal measurement whereas resistivity uses four terminals and is sensitive to a much smaller part of the sample. If the sample carrier density is inhomogeneous the two coefficients would tend to exhibit different phases. We checked for such problems by comparing the phase of the resistivity as measured by using the usual four-sample contacts with that obtained by using only the same two contacts as required for the thermopower. We found no significant phase differences over the relevant range of measurements, i.e. the range over which the thermopower oscillations were visible; in our most sensitive measurements this implies an inhomogeneity in the carrier density of < 0.2% over the whole sample.

For a singly occupied subband at a reasonably low field, the oscillations in the resistivity ρ_{xx}, which we denote by $\tilde{\rho}_{xx}$, are expected to obey (Ando 1974, Ando *et al* 1975, Isihara and Smrčka 1986)

$$\tilde{\rho}_{xx} = A(X/\sinh X)\exp(-2\pi^2 kT_D/\hbar\omega_c)\cos(2\pi f/B + \phi) \tag{1}$$

where A is a constant (Coleridge *et al* 1989), $X = 2\pi^2 kT/\hbar\omega_c$ (with the usual symbols) and T_D is the Dingle temperature which reflects the effect of impurity scattering on the width of the Landau levels. The carrier density n determines the frequency f via $f = n\pi\hbar/e$. The thermal damping factor $X/\sinh X$ causes an exponential reduction in the magnitude of $\tilde{\rho}_{xx}$ at high T/B so we have multiplied all our experimental $\tilde{\rho}_{xx}$ data by $\sinh X/X$ (using $m^* = 0.068\ m_e$ so that $2\pi^2 kT/\hbar\omega_c = T/B$ to within 0.1%) for presentation in the figures. According to equation (1) the resulting waveforms should be independent of the temperature at which the data were taken, and should show a smooth exponential decay as a function of B^{-1}. There is no theory for the phonon drag thermopower (see section 4) in the range of $kT/\hbar\omega_c$ of interest here. We have no reason to expect the thermopower oscillations $\tilde{S}_{xx}$ to obey equation (1) but we have still multiplied our $\tilde{S}_{xx}$ data by $\sinh X/X$ to emphasize the oscillations at high T/B.

Most of the data were recorded at equal intervals of $1/B$ (with typically 12–18 points per oscillation) which allowed filtering to be performed to separate the oscillations from the two subbands; filtering was done using the Fourier transforms and we used a pass band varying from 4–20 T at the lowest carrier density to 6–30 T for the saturated sample. Because the upper subband frequency was always very low it could not be separated from the background, but there was no difficulty in isolating the lower subband oscillations which are the focus of this work and are the ones shown in all the figures.

In his work on $\tilde{\rho}_{xx}$, Coleridge (1990) made use of the phase of the oscillations. As we mentioned in the introduction, we might expect the behaviour of the phase to be important in looking for a transition between intra- and inter-Landau scattering. Following Coleridge (1990) we chose a reference frequency f_R as close as possible to the observed frequency f; this choice was always made using $\tilde{\rho}_{xx}$ data at $T \sim 1$ K. These same data also allow us to fix the reference phase. To do this we simply take the last zero-crossing point of $\tilde{\rho}_{xx}$ (at the highest B) and choose a reference phase ϕ_R in equation (1) consistent with this. All other experimental data, taken under the same illumination conditions, use these same values of f_R and ϕ_R to define a local phase ϕ. We find all the zero-crossing points, say B_D, of the experimental data, calculate all the expected crossing points, say B_R, using equation (1) with f_R and ϕ_R, and finally define the local phase as $\phi = (B_D - B_R)f_R$; this procedure provides two points per oscillation.

As we shall see, the phase shows many interesting effects. We sometimes see abrupt phase changes (usually half a cycle); in these cases the numerical procedure tends to result in a negative phase change of half a cycle, but this is arbitrary to an integer number of cycles. Smoothly varying phase changes are not subject to the same arbitrariness.

3. Results

Most of the new phonomena are seen only when the two subbands are occupied. Fortunately, when cooled in the dark, the sample has no carriers in the upper subband and data taken under these conditions provide a useful basis for comparison with data taken when both subbands are occupied.

3.1. Single-band occupancy

A small selection of results on $\tilde{\rho}_{xx}$ for this case is shown in figure 1. As we mentioned in section 2 the waveforms in figure 1(a) have been multiplied by $\sinh X/X$ and, as would be expected if equation (1) is appropriate, the waveforms taken at different T then become essentially identical. The amplitude A in equation (1) is independent of T within experimental error and the exponential decays that are seen in figure 1(a) yield a Dingle temperature T_D of about 1.3 K. At $T \sim 4$ K the oscillations are visible with good signal to noise until $kT/\hbar\omega_c \sim 0.5$ but are below the noise level by 0.6. All the data have been truncated at low B at the point where the signals become obviously noisy. The phases shown in figure 1(b) exhibit no unexpected features. In our experience,

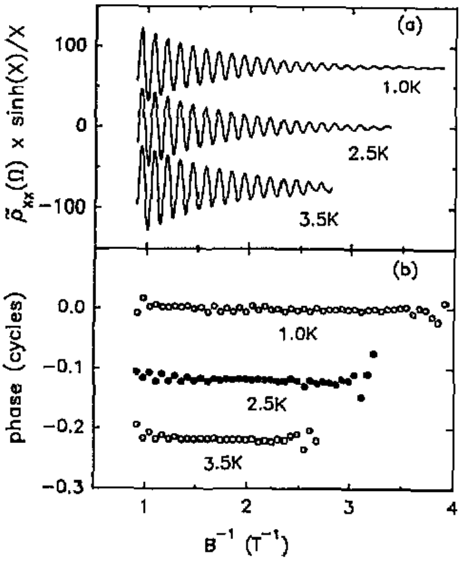

Figure 1. (a) Typical data showing the oscillations in the resistivity $\tilde{\rho}_{xx}$ as a function of inverse magnetic field when the sample has only one occupied subband. The frequency is very close to 8.00 T. The waveforms have all been multiplied by the inverse of the thermal damping factor to remove the temperature dependence and the data at 2.5 K and 3.5 K have been offset for clarity. At 1.0 K the zero field resistivity is 33.3 Ω and the mobility is 48.5 m^2 V^{-1} s^{-1}. (b) The phases of the oscillations shown in (a). The data at 2.5 and 3.5 K have been offset by −0.1 and −0.2 cycles for clarity.

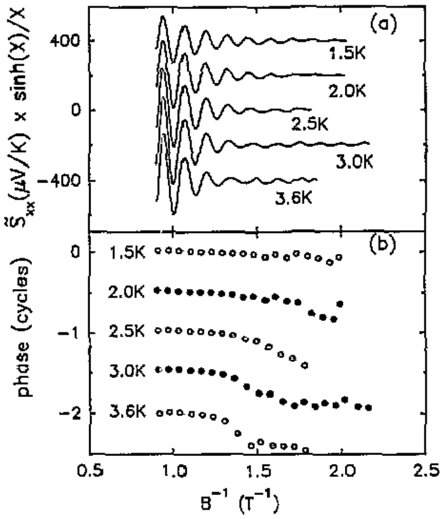

Figure 2. (a) The oscillations in the thermopower $\tilde{S}_{xx}$ as a function of inverse magnetic field for the sample in the same condition as appropriate to figure 1. The traces at successive temperatures have been offset by multiples of 200 μV K^{-1}. We have also multiplied the waveforms by the inverse of the thermal damping factor. (b) The phases of the oscillations shown in (a). Successive sets of data have been offset by multiples of 0.5 cycle.

the data in figure 1(a) and (b) are typical of those from a heterojunction with a single occupied subband and equation (1) is well obeyed (e.g. Fletcher *et al* 1990).

The equivalent data for $\tilde{S}_{xx}$ are shown in figure 2. We do not expect equation (1) to be valid though it does give an approximate description of the waveforms. If T_D is treated as a T dependent parameter (which of course is contrary to the spirit of equation (1)) then the values required to reproduce the data range from $T_D \sim 4$ K at $T = 1.5$ K to $T_D \sim 5.5$ K at $T = 3.5$ K. This shows that the amplitude $\tilde{S}_{xx}$ decays much more rapidly with B^{-1} compared to that for $\tilde{\rho}_{xx}$. In fact the oscillations in figure 2(a) cannot be followed beyond $kT/\hbar\omega_c \sim 0.33$, and the data reproduced there are from an experimental run which gave the best signal to noise ratio. In agreement with previous work (D'Iorio *et al* 1988) we find $\tilde{S}_{xx}$ and $\tilde{\rho}_{xx}$ to be in phase, to within 0.05 cycles, for $kT/\hbar\omega_c < 0.2$. However, at higher $kT/\hbar\omega_c$ we see a smooth phase shift which appears to saturate at −(0.40 to 0.45) cycles in the data shown.

The results on this part of the experiment can be summarized as follows. $\tilde{\rho}_{xx}$ obeys equation (1) in both magnitude and phase. $\tilde{S}_{xx}$ shows a dependence on T/B contrary to equation (1), unless T_D is taken to be a temperature dependent variable. The only new feature that we observe is the phase shift shown by $\tilde{S}_{xx}$ when $kT/\hbar\omega_c >$ 0.2.

3.2. Two-band occupancy

The large majority of our results were obtained on the two cases appropriate to the sample having just entered two-band occupancy, with an upper subband electron density $n_1 < 0.1 \times 10^{15}$ m^{-2}, and when it is saturated (i.e. shows no further change of electron density with illumination) with $n_1 \sim 0.3 \times 10^{15}$ m^{-2}. The quoted densities are run dependent so $\tilde{S}_{xx}$ and $\tilde{\rho}_{xx}$ can be compared only when both are from the same run and this is the case for all the data presented here.

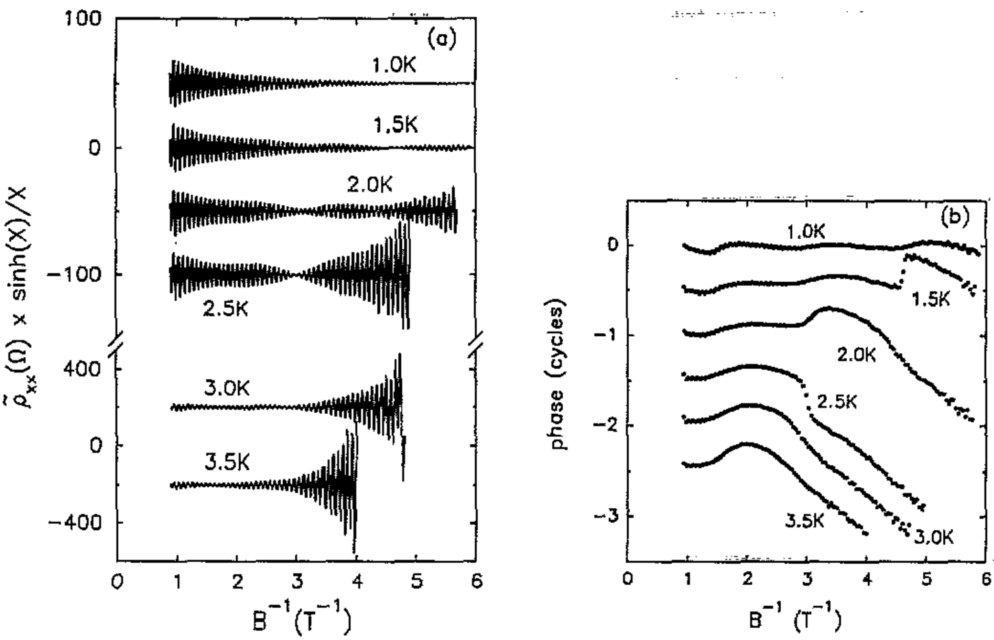

Figure 3. (a) The oscillations in the resistivity $\tilde{\rho}_{xx}$ as a function of inverse magnetic field for the sample in the saturated condition. All waveforms have been multiplied by the inverse of the thermal damping factor. Notice the scale change for the data at 3.0 K and 3.5 K. The traces have been offset to arbitrary zeros for clarity. The frequency at the higher fields is 13.0 T and at 1.0 K the resistivity of the sample is 10.4 Ω giving an average mobility of 90 m^2 V^{-1} s^{-1} (after including about 0.3 × 10^{15} m^{-2} electrons in the upper band). (b) The phases of the oscillation shown in (a). Successive data sets have been offset by multiples of −0.5 cycle.

We first examine the $\tilde{\rho}_{xx}$ results. Figure 3 shows the waveforms and phases for the saturated sample. The waveform at $T = 1.0$ K shows little amplitude modulation from the upper subband electrons and can be used to obtain an estimate of $T_{\mathrm{D}} \sim 0.7$ K. At $B^{-1} < 1.5$ T^{-1} for any T, the observed amplitude A in equation (1) is approximately a constant, i.e. in this field region the waveforms in figure 3(a) are very similar as expected from equation (1). Within a range determined by the conditions $B^{-1} >$ 1.5 T^{-1} and $kT/\hbar\omega_{\mathrm{c}} < 0.35$, the amplitude is seen to be modulated in a way similar to that described by Coleridge (1990) and Leadley *et al* (1989a), but in our case it is always quite weak. As noted by these authors, the modulation becomes stronger as T increases but the positions of the maxima and minima of A are fixed. In the data shown in figure 3(a) the positions of the modulation minima are approximately at $B^{-1} = 1.6, 3.1$ and $4.6\ \mathrm{T}^{-1}$ which correspond to an upper subband oscillation frequency $f_1 \sim 0.66$ T or carrier density $n_1 \sim 0.32 \times 10^{15}\ \mathrm{m}^{-2}$.

The significant new features in these data appear for $kT/\hbar\omega_c > 0.35$. In this range equation (1) becomes completely invalid and the amplitude A appears to increase very rapidly beyond this point. As can be seen in figure 3(a), at $B^{-1} \sim 4.0\ \mathrm{T}^{-1}$ and $T = 3.5$ K (which corresponds to $kT/\hbar\omega_c \sim 0.71$) the actual waveform of $\tilde{\rho}_{xx}$ is over two orders of magnitude larger than we would have expected if we had used the data at $T = 1.0$ K and equation (1) as a guide. This anomalous amplitude dependence is so extreme that at 4 K we can see well resolved oscillations to $kT/\hbar\omega_c \sim 0.80$ whereas in the single-band case the limit is about 0.50.

The phase information in figure 3(b) is also intriguing. For $kT/\hbar\omega_c < 0.35$ the phase is relatively constant (c.f. the whole of the data set taken at 1.0 K). Some structure is visible which is most noticeable at the positions of the amplitude minima mentioned above. Here the phase shows a relatively rapid rise amounting to a total of 0.1–0.2 cycles. However, beyond $kT/\hbar\omega_c \sim 0.35$ the phase for all data always shows a smooth decline which is close to linear in B^{-1}, the slope being -0.60 T in the data shown and independent of T. This slope is the same (to within about 10%) as the frequency f_1 of the oscillation from the upper subband. Another way of stating the same information is that the observed frequency in this region is $f_0 - f_1$ (f_0 being the lower subband frequency) rather than f_0. Coleridge (1990) mentions a similar effect but he does not associate it with the anomalous amplitude increase that we observe in the same region. Because the onset of the anomalous amplitude and phase dependence is controlled by $kT/\hbar\omega_c$, it moves to higher B as T increases. As it does so it moves through the modulation minima and appears to be responsible for the zero amplitude at $kT/\hbar\omega_c \sim 0.35$ noted above, and also produces an associated rather abrupt phase shift of 0.5 cycles seen in figure 3(b). As already noted, once $kT/\hbar\omega_c$ exceeds 0.45, all amplitude minima and associated phase discontinuities disappear.

$\tilde{\rho}_{xx}$ results for the case when the upper subband is only just occupied are shown in figure 4(a) and (b). Again, at $B^{-1} < 1.5\ \mathrm{T}^{-1}$ the amplitude A seems to be independent of T and above $kT/\hbar\omega_c \sim 0.35$ the amplitude becomes T dependent and is anomalously large. This amplitude increase is associated with a phase decrease consistent with $f_1 \sim 0.1$ T or $n_1 \sim 0.05 \times 10^{15}\ \mathrm{m}^{-2}$. We see no distinct oscillations from the upper subband and this is in accord with the small value of n_1. Although the effects in figure 4 are weaker than those in figure 3, they are consistent.

With two subbands occupied the behaviour of $\tilde{S}_{xx}$ is quite different from that of $\tilde{\rho}_{xx}$ described above. Amplitude and phase data for the case of the sample just entering second-subband occupation are shown in figure 5(a) and (b). Each of the waveforms in figure 5(a) shows amplitude modulation with a minimum whose position is temperature dependent. The locations of the minima for this set of data are $B^{-1} = 1.95 \pm 0.05,\ 1.62 \pm 0.05,\ 1.40 \pm 0.05,\ 1.35 \pm 0.010,\ 1.29 \pm 0.10$ and $1.30 \pm 0.10\ \mathrm{T}^{-1}$ for temperatures of 1.5, 2.0, 2.6, 3.0, 3.5 and 3.9 K respectively. The locations vary somewhat from run to run and presumably depend on n_1 but certainly do not scale as $kT/\hbar\omega_c$. Associated with each minimum is a phase shift (figure 5(b)). At low T the amplitude goes to zero at the minimum and the phase shift, which occurs at the same point, is abrupt and amounts to 0.5 cycle. At higher T there is no zero in the amplitude but a phase shift of 0.5 cycle still occurs, albeit spread over a number of cycles. At the positions of the minima there is no analogous behaviour seen in $\tilde{\rho}_{xx}$.

The saturated sample shows a more complex behaviour of $\tilde{S}_{xx}$, figure 6(a) and (b), but one which has some features in common with that described above. At 2.6 K two

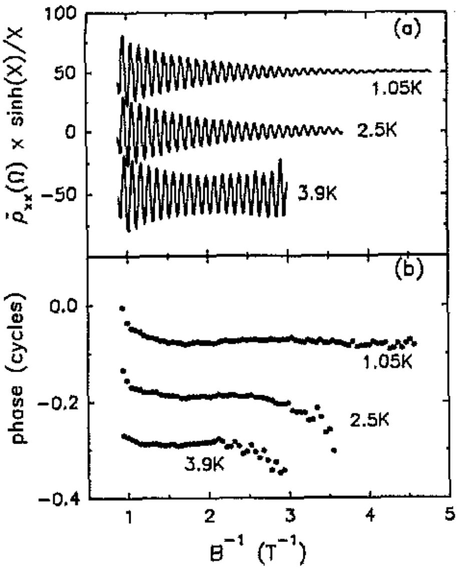

Figure 4. (a) The oscillations in the resistivity $\tilde{\rho}_{xx}$ as a function of inverse magnetic field for the sample in which the second subband is only just occupied. All waveforms have been multiplied by the inverse of the thermal damping factor. The frequency is 9.80 T and the zero-field resistivity at 1.0 K is 18.9 Ω giving a mobility of 70 m^2 V^{-1} s^{-1}. (b) The phases of the oscillations shown in (a). Successive data sets have been offset by multiples of −0.1 cycle.

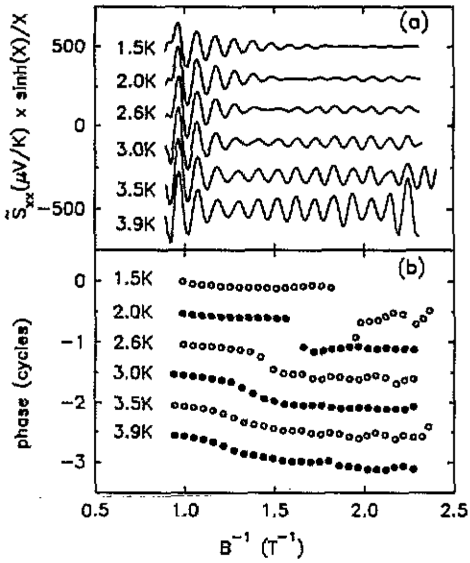

Figure 5. (a) The oscillations in the thermopower $\tilde{S}_{xx}$ as a function of inverse magnetic field for the sample in the same condition as is appropriate to figure 4. All the waveforms have been multiplied by the inverse of thermal damping factor. (b) The phases of the data shown in (a) with each successive data set offset by a multiple of −0.5 cycle.

strong modulation minima are evident and each is associated with an abrupt phase shift of 0.5 cycle. The two minima show a different behaviour as a function of T. The one at higher B has a location which is clearly T dependent in a manner similar to the case when the second subband is barely occupied though all the locations have now moved to higher fields. However, at $T = 2$ K this minimum is weak and by 1.5 K it has disappeared; in both these cases there are no strong phase changes to be seen. At 3.0 and 3.5 K the minimum is clearly visible, as is the associated phase change, though by 3.5 K it is moving out of the range of these data and it is not clear if the phase change is still 0.5 cycles.

The other minimum at lower field ($B^{-1} \sim 2.3$ T^{-1} in these data) is visible over the range 2.0–3.5 K and its location appears to be independent of T and coincident with a modulation *maximum* in $\tilde{\rho}_{xx}$. By 1.5 K there is only a weak minimum and this has no large phase shift associated with it. The location of this minimum varies from run to run and clearly depends on the upper subband electron density. However, the qualitative features noted here remain unchanged.

4. Discussion

Most of the new features that we observe seem to be triggered by, or at least accentuated by, occupation of the second subband and comprise:

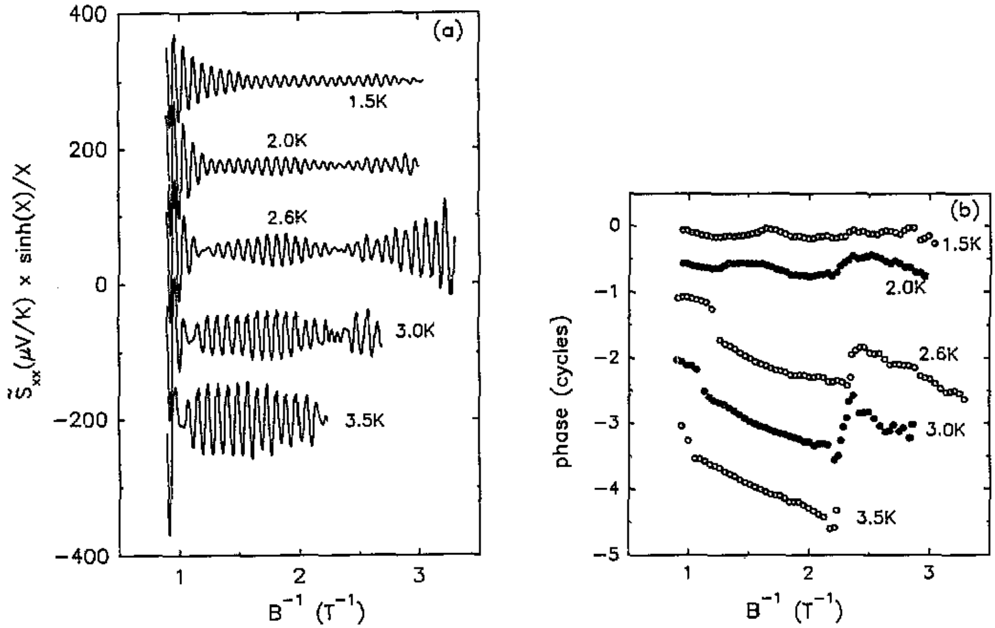

Figure 6. (a) The oscillations in the thermopower $\tilde{S}_{xx}$ as a function of inverse magnetic field for the sample in the same condition as is appropriate to figure 3. All waveforms have been multiplied by the inverse of the thermal damping factor. (b) The phases of the oscillations shown in (a). The data at 2.0 K, 2.5 K, 3.0 K and 3.5 K have been offset by −0.5, −1.0, −2.0 and −3.0 cycles respectively.

(i) the anomalous amplitude increase of $\tilde{\rho}_{xx}$ beyond $kT/\hbar\omega_c \sim 0.35$ (figure 3(a)) and the associated steady phase decrease which is linear in B^{-1} (figure 3(b)); the mechanism responsible for this seems to interfere with the usual oscillatory amplitude modulation (i.e. caused by the Landau levels of the upper subband passing through ε_F) in such a way that when an oscillatory minimum is near $kT/\hbar\omega_c \simeq 0.35$, the amplitude goes to zero and there is an associated abrupt 0.5 cycle phase change.

(ii) the amplitude minimum, seen in $\tilde{S}_{xx}$ (figure 5(a) and 6(a)) in the range B^{-1} =0.9–1.6 T^{-1} which has a T dependent location, and which has no counterpart in $\tilde{\rho}_{xx}$; there is a phase shift of 0.5 cycles associated with this feature (figure 5(b) and 6(b)) and under some conditions this phase shift is abrupt.

(iii) the subsequent amplitude minimum and phase reversal seen in $\tilde{S}_{xx}$ for the saturated sample (near $B^{-1} = 2.3$ T^{-1} for the data reproduced in figure 6(a) and(b)); the location of this minimum is independent of T and corresponds to a *maximum* in the amplitude modulation of $\tilde{\rho}_{xx}$.

In addition there is the phase shift that is observed in $\tilde{S}_{xx}$ for the sample with only a singly occupied subband (figure 2(a)).

We believe that the presence of *inelastic* inter-Landau level electron scattering is responsible for most, if not all, of these phenomena. In the absence of a quantitative theory for the effect of this type of scattering on $\tilde{\rho}_{xx}$ and $\tilde{S}_{xx}$, we attempt to understand the main features of the data in a qualitative way.

We first focus on $\tilde{S}_{xx}$. The oscillations we see are almost certain to be due only to phonon drag (c.f. Fletcher *et al* 1990) which is controlled by inelastic phonon scattering. It is known that diffusion thermopower will show phase shifts relative to $\tilde{\rho}_{xx}$ of up to 0.5 cycles (Havlová and Smrčka 1986), but in the present case the monotonic diffusion thermopower probably never exceeds 10 μV K^{-1} in magnitude. However, the magnitude of the oscillations that we observe never drops significantly below 1 μV K^{-1} (and this only at the highest $kT/\hbar\omega_c$) and so to identify them as due to diffusion implies that the oscillations in the diffusion thermopower are at least as large as the monotonic parts (and at a low $kT/\hbar\omega_c$ considerably larger). We believe that this is inconsistent with the current theory. These statements can probably be made completely convincing only by examining $\tilde{S}_{xx}$ at much lower temperatures where phonon drag effects become small. (This is especially true when one compares the situation with that for bulk metals where $\tilde{S}_{xx}$ is known to be due only to diffusion effects (Fletcher 1983).) Meanwhile we proceed with the assumption that $\tilde{S}_{xx}$ is completely dominated by phonon drag and hence inelastic scattering of the electrons.

The behaviour in (ii) above suggests that we are observing two contributions to $\tilde{S}_{xx}$ which have different dependences on T/B and which are essentially in antiphase. We believe that these two contributions are to be identified with inelastic *intra*-Landau level scattering, dominant at low $kT/\hbar\omega_c$, and inelastic *inter*-Landau level scattering, dominant at high $kT/\hbar\omega_c$. The observed transition point, i.e. the position of the modulation minimum in $\tilde{S}_{xx}$, takes place in the range $kT/\hbar\omega_c$ =0.15–0.25, which is rather low for inter-Landau level scattering within a single band if we accept the dominant phonon mode argument of Kent *et al* (1988) as applied to heat pulses; this argument equates the energy of the dominant phonon mode in the heat pulse ($2.8kT_{\mathrm{ph}}$) with that of the Landau level separation to give $kT_{\mathrm{ph}}/\hbar\omega_c = 0.35$†. In our case the presence of the second subband Landau levels means that there is always a pair of levels near ε_F that lie within $\frac{1}{2}\hbar\omega_c$ of each other suggesting that the appropriate condition might be relaxed to $kT/\hbar\omega_c \sim 0.18$ which is then similar to the experimental values.

The second amplitude minimum in $\tilde{S}_{xx}$ described in (iii) supports this identification of inelastic inter-Landau level scattering being dominant at high $kT/\hbar\omega_c$. The fact that the modulation in $\tilde{S}_{xx}$ is in antiphase with that in $\tilde{\rho}_{xx}$ indicates that the mechanism responsible is at a *minimum* when an upper subband Landau level is coincident with ε_F. This is what we would expect for inter-Landau scattering and opposite to that expected from intra-Landau level scattering which is presumed to control $\tilde{\rho}_{xx}$ in this range.

Because the two mechanisms give oscillations in antiphase, the crossover point where the dominant mechanism switches from one to the other should be indicated by an abrupt phase change of 0.5 cycle. This is generally in accord with the observations described in (ii). Under some circumstances, e.g. figure 5(b) at higher tempertures, the phase changes slowly with field though the overall change is always consistent with 0.5 cycle. This might suggest that the two mechanisms are not always exactly in antiphase but a more detailed model would be needed to explain these features. The

† One of the referees has pointed out the following subtle point. In the heat pulse experiments the phonon energy distribution is taken to be that of a black body with a spectrum $\sim x^3/(e^x - 1)$ where $x = h\nu/kT_{\mathrm{ph}}$, ν being the phonon frequency and T_{ph} the temperature of the heater; this peaks at $x \sim 2.8$ giving $kT_{\mathrm{ph}}/\hbar\omega_c \sim 0.35$ in the experiments. In the case of $\tilde{S}_{xx}$ it might be argued that the heat current is the relevant quantity and with a fixed phonon mean free path the appropriate spectrum is that of the specific heat rather than the energy, i.e. $x^4e^x/(e^x - 1)^2$; this peaks at $x = 3.8$ giving $kT/\hbar\omega_c \sim 0.26$. Of course this point is not relevant to $\tilde{\rho}_{xx}$.

absence of any strong phase changes in $\tilde{S}_{xx}$ for the saturated sample at 1.5 and 2.0 K (figure 6(a)) suggests that inter-Landau level scattering is never dominant under these conditions though the amplitude modulation is consistent with its presence in both cases.

It seems reasonable to associate the phase shifts observed in $\tilde{S}_{xx}$ for the sample with only a singly occupied band (figure 2(b)) to the same transition from inelastic intra- to inelastic inter-Landau level scattering though it would be more convincing if the phase changes were abrupt. We should also point out that we have examined other samples which showed no strong phase change in the region of interest. Also the phase shift appears near $kT/\hbar\omega_c \sim 0.25$ which is rather low for the dominant mode argument to be convincing (see the previous footnote which shows that a modified argument can predict a value of $kT/\hbar\omega_c$ closer to that observed). On the other hand the amplitude of the oscillations in $\tilde{S}_{xx}$ is particularly small for this sample, which may be a result of the high mobility and consequent small Landau level broadening. This would tend to reduce the contribution from inelastic intra-Landau level scattering and so perhaps increase the likelihood of observing inter-Landau level scattering. This possibility must be considered speculative but might be checked by measuring $\tilde{S}_{xx}$ for samples with a wide range of mobilities.

If inelastic inter-Landau level scattering between two subbands were the dominant mechanism for $\tilde{S}_{xx}$ at high $kT/\hbar\omega_c$, we would expect the phase of the oscillations to be controlled by the *relative* positions of the two sets of Landau levels with respect to each other. This implies that we would see an effective frequency of $f_0 - f_1$ in this region but the data are more consistent with $f_0 - xf_1$ where x lies between 1 and 2. An observation that suggests this discrepancy is real is that between the points where the phase shift makes abrupt 0.5 cycle transitions (figure 6(b) for the data at $T = 2.5, 3.0, 3.5$ K) the phase decreases by close to one complete cycle within experimental error. We believe that the position of the phase discontinuity at low B ($B^{-1} \sim 2.3\ \mathrm{T}^{-1}$) is independent of T, but the other one is not. If this is so then the phase as a function of B^{-1} must have a slope which depends on T suggesting an alternative explanation must be found.

We now turn to the behaviour of $\tilde{\rho}_{xx}$. When $kT/\hbar\omega_c < 0.3$ the oscillations are presumed to result from both intra- and inter-subband *elastic* scattering (Coleridge 1990). In this region equation (1) serves as a first approximation to the observed behaviour and Coleridge has shown that the increasing amplitude modulation as the temperature rises is consistent with this mechanism. When $kT/\hbar\omega_c > 0.35$ the behaviour is quite different. The amplitude A (assuming equation (1) to be applied) rises by orders of magnitude and amplitude modulation is suppressed. Furthermore there is a continuous phase shift indicating an actual frequency close to $f_0 - f_1$. All of these features imply that a different physical mechanism is responsible for the oscillations. We tentatively identify this with inelastic inter-Landau level scattering. A feature which does support this identification is the appearance of the zero amplitude whenever an amplitude minimum coincides with $kT/\hbar\omega_c \simeq 0.35$. Such a behaviour could result from elastic inter-subband scattering being at a minimum here, via the Coleridge mechanism, so that inelastic inter-Landau level scattering (which is in antiphase) can become dominant at this point. As the temperature is increased still further, the latter mechanism presumably increases the amplitude A very rapidly and so the minimum is no longer visible. According to our previous arguments for $\tilde{S}_{xx}$ we would have expected the minimum to become a maximum and if one looks closely at the data for $\tilde{\rho}_{xx}$ at 3.5 K, figure 3(a), there is in fact evidence for a maximum superimposed on the rapid increase, but this is weak and not completely convincing.

In principle, hot electron studies could distinguish the presence of elastic and inelastic scattering in $\tilde{\rho}_{xx}$. If the former was appropriate, the oscillations would depend only on the temperature of the electrons and not on that of the lattice. In the latter case both temperatures would be relevant. Interestingly Leadley *et al* (1989b) have published results on hot electron experiments for two samples in which the second subbands are occupied. They show that for $kT/\hbar\omega_c > 0.30$ there is an apparent increase in the electron energy loss rate which they ascribe to cyclotron phonon emission (i.e. phonons of energy $\hbar\omega_c$ being emitted as the electrons cool). However, our data show that the thermal damping factor $X/\sinh X$ is invalid in this situation and, if used, would lead to much lower apparent temperatures being obtained than really exist. Thus an alternative explanation of their results is that they have also seen the anomalous amplitude increase and misinterpreted it as indicating a lower temperature. We should point out that a single sample in which only the lower subband was occupied was also investigated by them and indicated a similar behaviour; we have not seen any anomalous amplitude increase at high $kT/\hbar\omega_c$ for such samples and we believe the thermal damping factor to be appropriate in this case. Clearly the area needs clarification.

Another experiment on which the present results could have a bearing is that of Blom *et al* (1990) who report the thermal damping factor to be inaccurate in their sample. They analysed their results in terms of an apparent variation of the effective mass m^* which had to be used in order to force the thermal damping factor to produce the same temperatures as those actually measured. They found that beyond $kT/\hbar\omega_c \sim 0.30$ the apparent effective mass was a function of $kT/\hbar\omega_c$ and had decreased to about 40% of its usual value by $kT/\hbar\omega_c \sim 0.6$. Again, this is consistent with the anomalous amplitude that we find in this region when the second subband is occupied. Since Blom *et al* do not show any original data it is not clear if the second subband is occupied, but this is a possibility in view of the fact that the electron density of their sample is $4.5 \times 10^{15}\ \mathrm{m}^{-2}$.

5. Conclusions

Many new phenomena have been found in the present investigation into the quantum oscillations in the thermopower and resistivity. Because the thermopower in the ^{4}He temperature range is primarily due to phonon drag, it is an ideal probe to examine inelastic electron–phonon scattering. The various amplitude and phase effects that we have seen in the thermopower oscillations seem to have a natural explanation in terms of a transition from inelastic intra-Landau level scattering at low $kT/\hbar\omega_c$ to inelastic inter-Landau level scattering at high $kT/\hbar\omega_c$. There seems little doubt that these mechanisms exist and the question was basically whether the experimental results were consistent with those that might be expected. Although there are some details which are not clear, the overall conclusion is that the data and the proposed mechanisms are indeed consistent. A model calculation would be required to be able to say more than this.

The new effects observed in the resistivity are probably also consistent with a transition from elastic intra- and inter-Landau level scattering at low $kT/\hbar\omega_c$ to inelastic inter-Landau level scattering at high $kT/\hbar\omega_c$ but the picture is less clear in this case.

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References

Ando T 1974 *J. Phys. Soc. Japan* **37** 1233
Ando T, Matsumoto Y and Uemura J 1975 *J. Phys. Soc. Japan* **39** 279
Blom F A P, Fontein P F, Wolter J H, Peeters F M, Wu X, Greeninckx F and Devreese J T 1990 *Surf. Sci.* **229** 70
Challis L J, Kent A J and Rampton V W 1989 *High Magnetic Fields in Semiconductor Physics II* ed G Landwehr (Berlin: Springer) p 529
Coleridge P T 1990 *Semicond. Sci. Technol.* **5** 961
Coleridge P T, Stoner R and Fletcher R 1989 *Phys. Rev.* B **39** 1120
D'Iorio M, Stoner R and Fletcher R 1988 *Solid State Commun.* **65** 697
Fletcher R 1983 *Phys. Rev.* B **28** 1721, 6670
Fletcher R, D'Iorio M, Harris J J and Foxon C T 1990 *Semicond. Sci. Technol.* **5** 1136
Fletcher R, Harris J J and Foxon C T 1991 *Semicond. Sci. Technol.* **6** 54
Hardy G A, Kent A J, Rampton V W and Challis L J 1989 *High Magnetic Fields in Semiconductor Physics II* ed G Landwehr (Berlin: Springer) p 537
Havlová H and Smrčka L 1986 *Phys. Status Solidi* b **137** 331
Isihara A and Smrčka L 1986 *J. Phys. C: Solid State Phys.* **19** 6777
Kent A J, Rampton V W, Newton M I, Carter P J A, Hardy G A, Hawker P, Russell P A and Challis L J 1988 *Surf. Sci.* **196** 410
Kubakaddi S S, Butcher P N and Mulimani B G 1989 *Phys. Rev.* B **40** 1377
Leadley D R, Nicholas R J, Harris J J and Foxon C T 1989a *Semicond. Sci. Technol.* **4** 885
—— 1989b *Semicond. Sci. Technol.* **4** 879
Lyo S K 1989 *Phys. Rev.* B **40** 6458